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On the Distribution of Geomagnetic Storms in the Sunspot Cycle

Part I

The Distribution of Great Magnetic Storms in the Sunspot Cycle

By

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概　　要

太陽黒点週期における磁気嵐の年発生頻度の分布について，磁気嵐の型を考慮しない時，著者達は大きな磁気嵐が太陽黒点週期と注目すべき関係があることを見出した。即ち太陽黒点の年平均値を n とし，n の前年との差の絶対値を $|\Delta n|$ とすれば，柿岡地磁気観測所における K 指数の一日の合計が 30 或いは 35 より大きな磁気嵐の年発生頻度は

$$an + b\,|\Delta n| + c$$

によってあらわされる。ここに a，b，c はそれぞれ最小自乗法によって求められた常数である。

§ 1. Introduction

Statistical investigations of correlations between annual means of magnetic and solar activity were made hitherto by many investigators [1] [2]. So it is well known that the frequencies of occurrence of magnetic storms have a fairly good correlation with sunspot numbers. In most cases of the statistics, however, maximum ranges of the horizontal component or other elements of geomagnetic field, magnetic character figure and u-measure were used as a measure of magnetic activity. In this note it is tried from the angle of K-indices to show the clear relationship between annual sunspot numbers and the annual frequencies of occurrence of great magnetic disturbance at Kakioka.

§ 2. Data of Geomagnetic Storms

For this statistics were used data of principal magnetic disturbances observed at Kakioka during the period from 1924 to 1951 [3] and geomagnetic indices K at Kakioka. Since magnetic indices K at Kakioka are available from 1942, we cannot but adopt data from 1942 to 1951 in order to see the distribution of geomagnetic disturbances over the indices K at Kakioka.

In the first place, let N $(\Sigma K \geq i)$ denotes the annual numbers of magnetic disturbances whose maximal daily sum of indices K during each disturbance is equal to or more than i.

During the interval above mentioned, 384 principal magnetic disturbances are available without regard to their type and magnitude. Annual distributions of them for the various ranges of ΣK (daily sum of indices K) are given in Table I, and histogram of them is shown in Fig. 1. Table II shows the annual distributions of N $(\Sigma K \geq 30)$ and N $(\Sigma K \geq 35)$ from 1942 to 1951. Fig. 2 shows the annual distribution of total number of magnetic disturbances, N $(\Sigma K \geq 30)$ and N $(\Sigma K \geq$

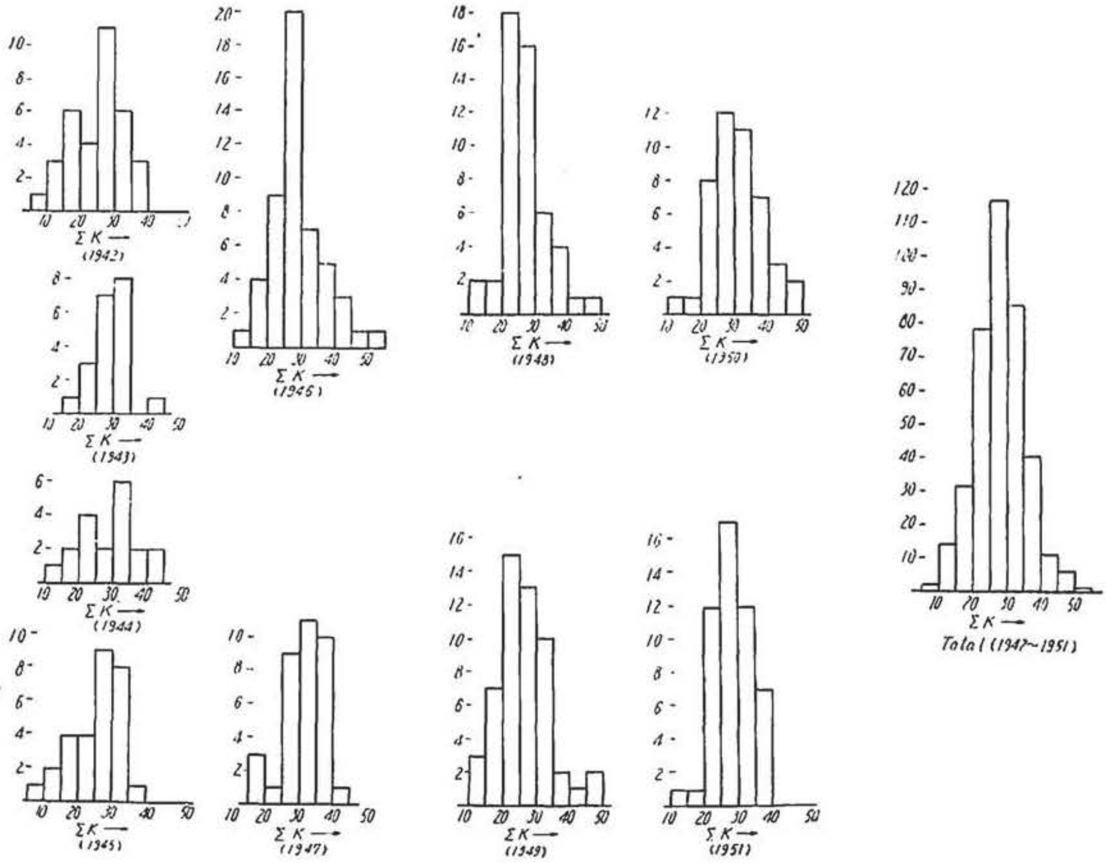

Fig. 1. Distribution of geomagnetic disturbances from 1942 to 1951.

Table I. Annual distribution of geomagnetic disturbances from 1942 to 1951 at Kakioka

Range of ΣK	1942	1943	1944	1945	1946	1947	1948	1949	1950	1951	Sum
5~ 9	1	0	0	1	0	0	0	0	0	0	2
10~14	3	0	1	2	1	0	2	3	1	1	14
15~19	6	1	2	4	4	3	2	7	1	1	31
20~24	4	3	4	4	9	1	18	15	8	12	78
25~29	11	7	2	9	20	9	16	13	12	17	116
30~34	6	8	6	8	7	11	6	10	11	12	85
35~39	3	0	1	1	5	10	4	2	7	7	40
40~44	0	1	1	0	3	1	1	1	3	0	11
45~49	0	0	0	0	1	0	1	2	2	0	6
50~54	0	0	0	0	1	0	0	0	0	0	1
Sum	34	20	17	29	51	35	50	53	45	50	384

Table II. Annual distribution of N ($\Sigma K \geq 30$) and N ($\Sigma K \geq 35$) from 1942 to 1951.

	1942	1943	1944	1945	1946	1947	1948	1949	1950	1951	Sum
N ($\Sigma K \geq 30$)	9	9	8	9	17	22	12	15	23	19	85
N ($\Sigma K \geq 35$)	3	1	2	1	10	11	6	5	12	7	40

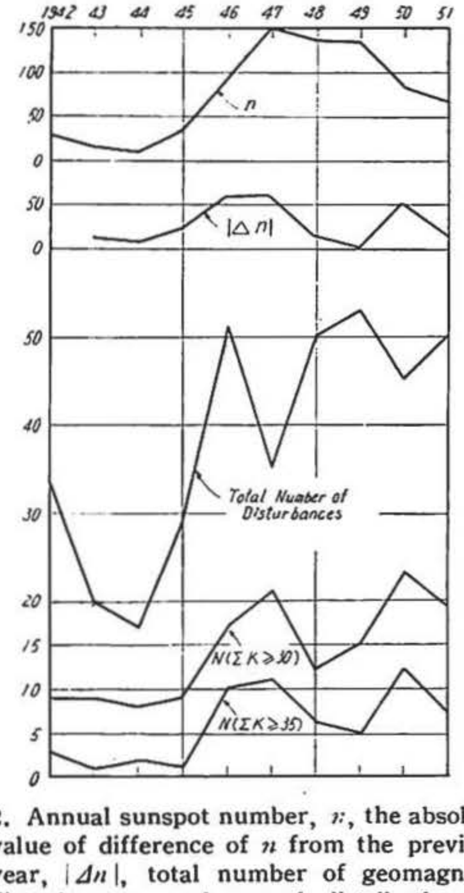

Fig. 2. Annual sunspot number, n, the absolute value of difference of n from the previous year, $|\Delta n|$, total number of geomagnetic disturbances, and annual distributions of N ($\Sigma K \geq 30$) and N ($\Sigma K \geq 35$).

35), together with annual sunspot numbers, n, and $|\Delta n|$ which represent the absolute value of difference of n from the previous year.

§ 3. Analysis and result

From Table II and Fig. 2, it is found that both N ($\Sigma K \geq 30$) and N ($\Sigma K \geq 35$) have peaks over the solar cycle, that is, in the period of ascent and especially of decline of sunspot number n rather than around its maximum, showing a close connection with $|\Delta n|$, not only n. Therefore, it will be able to presume the following regression formula :

$$an + b|\Delta n| + c = n(\Sigma K \geq i) \qquad \cdots\cdots\cdots\cdots (1)$$

where a, b and c are coefficients determined by the method of least square, and the fairly large numerical values of i, such as 30 or 35, are suitable for this formula.

When annual sunspot numbers are given, if these three coefficients, a, b, c, in eq. (1) are determined, predicted values of N $(\Sigma K \geq 30)$ or N $(\Sigma K \geq 35)$ for each year, denoted by N_{calc} $(\Sigma K \geq 30)$ or N_{calc} $(\Sigma K \geq 35)$ respectively, are able to be obtained from eq. (1).

Probable values of these coefficients, a_0, b_0, c_0, for the cases of N $(\Sigma K \geq 30)$ and N $(\Sigma K \geq 35)$ are determined as shown in Table III.

Table III. Probable values of coefficients in eq. (1)

Probable values			median error	
a_0	b_0	c_0	r	
0.052	0.128	7.213	3.1	for the case of N $(\Sigma K \geq 30)$
0.036	0.117	0.006	1.8	for the case of N $(\Sigma K \geq 35)$

Thus, the results of calculation of N_{calc} $(\Sigma K \geq 30)$ and N_{calc} $(\Sigma K \geq 35)$ from 1943 to 1951 are displayed in Fig. 3. Comparing these figures with curves of N $(\Sigma K \geq 30)$ and N $(\Sigma K \geq 35)$ in Fig. 2, we see a fairly good fitness of calculations by eq. (1) to observations. Now, if we let r_N $(\Sigma K \geq i)$ N_{calc} $(\Sigma K \geq i)$ represent the correlation coefficient between observation number N $(\Sigma K \geq i)$ and calculation N_{calc} $(\Sigma K \geq i)$, correlation coefficients for $\Sigma K \geq 30$ and $\Sigma K \geq 35$ are obtained respectively as follows :

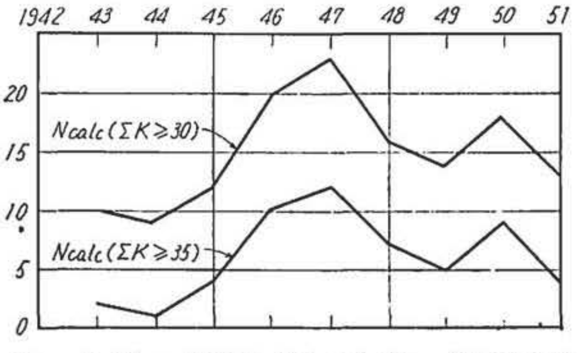

Fig. 3. N_{calc} $(\Sigma K \geq 30)$ and N_{calc} $(\Sigma K \geq 35)$.

$$r_N\ (\Sigma K \geq 30)\ N_{calc}\ (\Sigma K \geq 30) = 0.82$$
$$r_N\ (\Sigma K \geq 35)\ N_{calc}\ (\Sigma K \geq 35) = 0.89$$

Whereas, simple correlation coefficients between N $(\Sigma K \geq i)$ and sunspot number n itself, $r_N\ (\Sigma K \geq i).\ n$, are not so good as follows :

$$r_N\ (\Sigma K \geq 30).\ n = 0.61$$
$$r_N\ (\Sigma K \geq 35).\ n = 0.67$$

From above results, we find that there is a fairly good agreement between calculated values of eq. (1) and observations, although the statistics are not so sufficient yet to establish a conclusive characteristics of distribution of magnetic storms in the sunspot cycle.

As an example, if anyone wish to know, he may predict the number of great magnetic storms, $N_{calc}\ (\Sigma K \geq 30)$ or $N_{calc}\ (\Sigma K \geq 35)$, in 1957 as follows :

$$N_{calc}\ (\Sigma K \geq 30) \sim 22$$

and

$$N_{calc}\ (K\Sigma \geq 35) \sim 11$$

where the predicted value of sunspot number, n, for 1957 is assumed about 170, as calculated by using a so-called basic function in Part II, of our paper under the same title.

It is desired that similar analysis are carried out by using extended data over several solar cycles and, furthermore, K_p instead of K at Kakioka to test the above conclusion.

It seems, at any rate, that solar activity, relating to the ejection of corpuscular streams which cause great magnetic storms on the earth, more increases in the period of ascent or decline of the sunspot numbers rather than around the sunspot maximum.

We wish to thank the members of Kakioka Magnetic Observotory and Geoelectric and Geomagnetic Laboaratory of Meteorological Research Institute, Tokyo, for their helpful criticism in this work,

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Part II

概　　要

太陽黒点周期と急始磁気嵐の統計的研究から，IGY 観測期間中における 1957 年及び 1958 年の柿岡の急始磁気嵐の年発生頻度は，それぞれ 80 %の信頼度において 30 ± 4 及び 29 ± 4 の範囲で予報出来ることを示す．

§ 1. Introduction

It has been well known [1] that there is a close connection between annual frequencies of occurrence of magnetic storms with sudden commencement (SC storms) and sunspot numbers. Apart from this outstanding fact, the authors first investigated annual frequencies of occurrence of great magnetic storms, N, in the sunspot cycle without regard to type of storms, i. e. which include non-SC storms, and found that N is calculated by following formula,

$$N = an + b\,|\Delta n| + c$$

where n denotes the annual sunspot number, $|\Delta n|$ the absolute value of difference of n from the previous year, and a, b and c constants to be determined by the method of least square.

In this paper Part II, authors investigate that how to forecast the annual frequencies of occurrence of SC storms (exclude non-SC storms) with in any range of error. And then authors estimate the frequencies of occurrence of SC storms in 1957 and 1958, the period of IGY observation.

§ 2. The correlation coefficient and the regression line

Table 1 shows the frequencies (%) of maximum ranges of SC storms for their successive 50 γ intervals which were observed during 33 years from 1924 to 1956 at Kakioka.

Table 1. The frequencies (%) of maximum ranges of SC storms during 33 years from 1924 to 1956 at Kakioka

	Maximum range							Total
H	<50	50~100	100~150	150~200	200~250	250~300	300γ <	
	1	36	34	12	8	4	5	100%
D	< 5	5~10	10~15	15~20	20′ <			
	7	39	35	16	3			100%
Z	<50	50~100	100~150	150~200	200γ <			
	29	58	10	1	2			100%

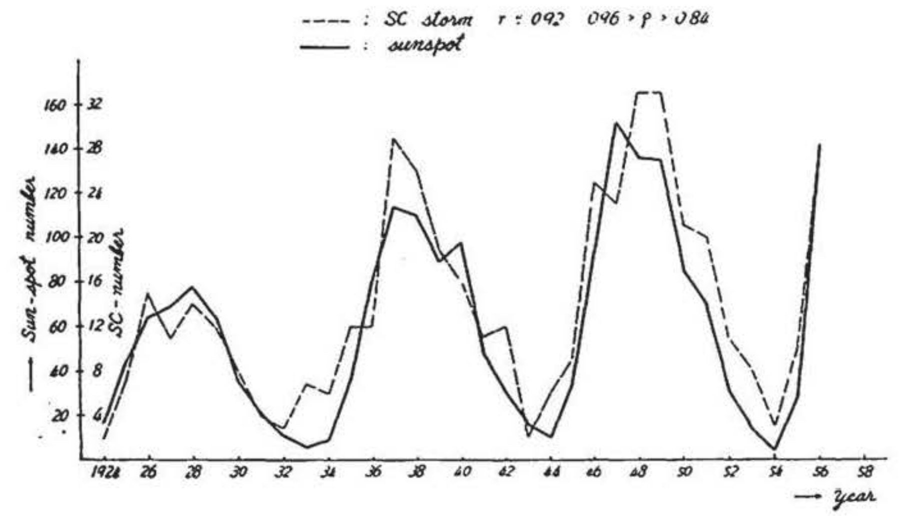

Fig. 1. The correlation between annual frequencies of SC storms and annual means of sunspot numbers

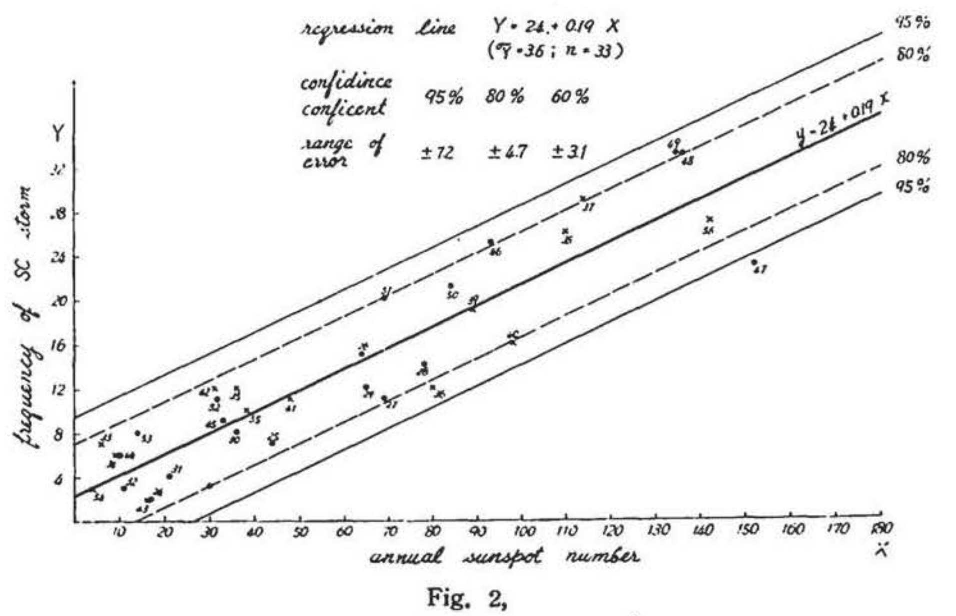

Fig. 2,

In Fig. 1 is graphically shown the correlation between annual frequencies of occurrence of SC storms and annual sunspot numbers. The correration coefficient of the samples, r is 0.92, and the range of correlation coefficient of populations ρ at the confidence coefficient of 95 % is $0.96 > \rho > 0.84$ Fig. 2. shows the regression line and the confidence belts at 95 % and 80 %. The regression equation is

$$Y = 2.4 + 0.19\,X \qquad (1)$$

where Y denotes annual frequencies of occurrence of SC storms and X annual sunspot numbers. The standard deviation of Y, σ_Y is calculated as 3.6.

Accordingly, it is concluded that if it is possible to forecast annual sunspot numbers, the forecasting of annual frequencies of occurrence of SC storms are also possible with the ranges of error, ± 7.2, ± 4.7 and ± 3.1 corresponding to the confidence coefficients of 95 %, 80 %, and 60 % respectively.

§ 3. The basic number

According to the investigation of Stewart and Panofsky [2], the annual sunspot numbers n are expressed as follows,

$$n = f\theta^a e^{-b\theta} \qquad (2)$$

where θ is the time variable measured from a minimum, and f, a and b are all parameters selected for any one cycle. Since the variable θ and three parameters f, a and b are all different in different cycles, it is difficult to estimate the future sunspot numbers, even though their formula can express approximately the annual sunspot numbers in the past.

In order to remove this difficulty, Granger [3] derived statistically a basic cycle which was represented only by one variable θ.

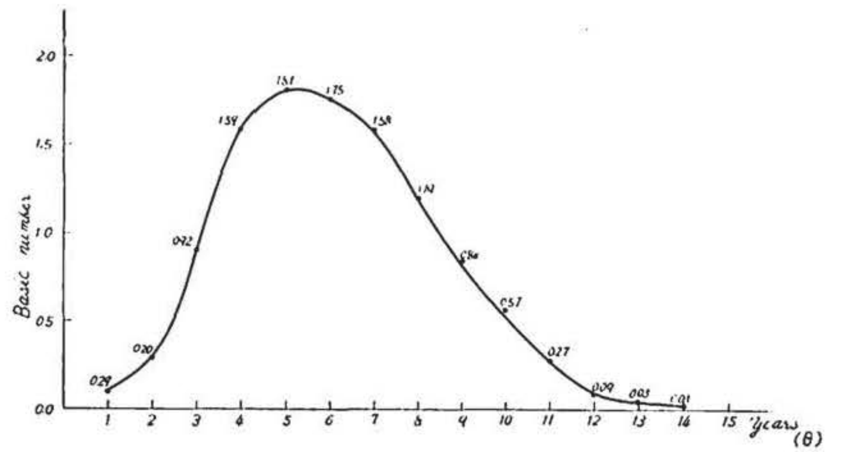

Fig. 3. The basic cycle (average curve of 18 cycles from 1755 to 1953)

Fig. 3 shows the basic cycle, which is derived by averaging over 18 cycles from 1755 to 1953, where the basic number represents the ratio of the annual mean to the respective cycle mean of sunspot numbers.

The cycle mean of sunspot number, M, as it is seen in Fig. 4, changes periodically during the 18 cycles, probably as a superposed result of two or more periodic changes. It may be not unreasonable to suppose from the figure $M > 55$ for the next cycle beginning from 1954.

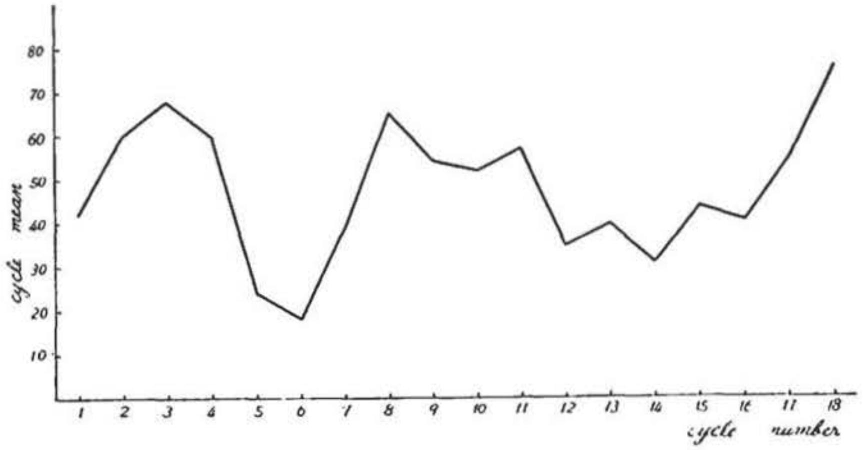

Fig. 4. Variations of cycle means in respet to sunspot cycles (18 cycles from 1755 to 1953)

In table 2 and 3 are given the frequency distributions of L and p, where p is the ratio of the length in months between a minimum and the next maximum to the length in months between two consecutive minima L.

Then, if we consider the case of $L \leq 123$ at $M > 55$ and $p \leq 0.42$ at $M > 45$ in table 2 and table 3, it will be estimated for the next cycle beginning from 1954 that the value of L is 10 years and the maximum falls in 1957, the fourth year reckoned from the minimum year 1954. Accordingly the following basic number is applicable.

Year	1954	55	56	57	58	59	60	61	62	63
basic number	0.29	0.92	1.59	1.81	1.75	1.58	1.19	0.84	0.57	0.27

Table 2. The relationship between the cycle means of sunspot numbers M and the length in months of each cycle from a minimum to the next minimum L

	$L \leq 123$	$123 < L \leq 136$	$L > 136$	Total
$M \leq 38$	0	0	4	4
$38 < M \leq 55$	2	4	2	8
$M > 55$	4	0	2	6
Total	6	4	8	18

Table 3. The relationship between the cycle mean of sunspot number M and $p = \frac{(\text{Max}) - (\text{Min})}{L}$

	$p \leq 0.42$	$p > 0.42$	Total
$M \leq 45$	3	6	9
$M > 45$	9	0	9
Total	12	6	18

§ 4. Application to forecasting and its results

Fig. 5. shows the relation between the basic number (x) and annual frequencies of occurrence of SC storms (y) during 10 years from 1947 to 1956. And the relationships is expressed by the least square as follows,

$$y = 19.0\,x - 4.1 \tag{3}$$

In this case the deviation of y, σ_Y is 2.85 and the belts of confidence limit at 95 % 80 % and 60 % are respectively $\pm$ 6.3, $\pm$ 3.9 and $\pm$ 2.5.

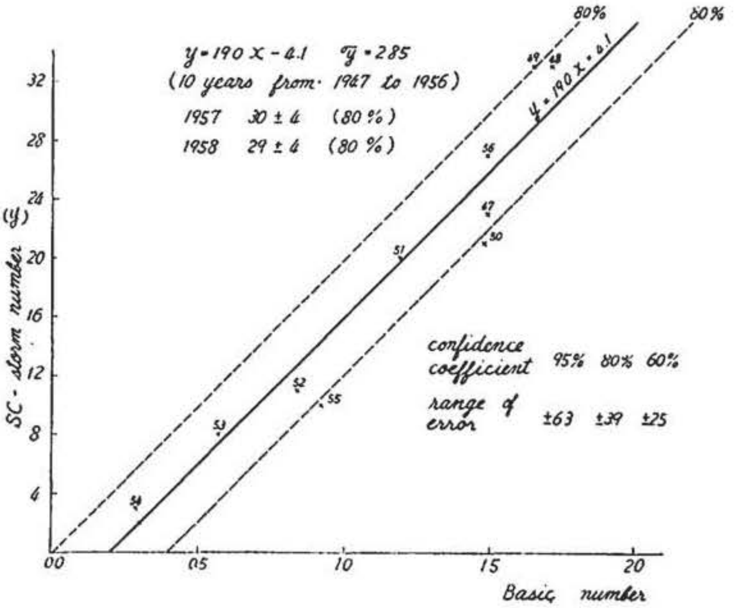

Fig. 5. The results of the forecast

From these results, annual frequencies of occurrence of SC storms in 1957 and 1958 are estimated to be 30 and 29, Considering the character of the basic curve, well fitted particularly over the middle range, and the striking increase of sunspot number up to date (the annual sunspot number for 1957 is assumed about 170), 34 instead of 30 may be more appropriately estimated for the annual frequencies of occurrence of SC storm in 1957, the former number being calculated by the uper limit of the belt of confidence at 80 %.

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